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A Mathematical Model of Fluid Flow in an Open Trapezoidal Channel with Lateral Inflow Channel

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Abstract

In this paper, an incompressible fluid flow in an open trapezoidal channel with one lateral inflow channel is investigated. The flow parameters that are investigated include the cross-sectional area, angle, length and velocity of the lateral inflow channel. The flow variables in the main trapezoidal channel include the depth and velocity of the fluid. These flow parameters in the lateral inflow trapezoidal channel are investigated on how varying each parameter independently affects the flow velocity in the main trapezoidal channel. The continuity and momentum equations are equations that govern this flow. Since these two equations are highly nonlinear, the finite difference method is used to approximate the solutions. The results are then presented by velocity profiles graphs and discussed. It is noted that a decrease in the cross-sectional area leads to an increase in the flow velocity and an increase in the length of the lateral inflow channel leads to a decrease in the flow velocity. It is also noted that an increase in the velocity of the lateral inflow channel leads to an increase in the flow velocity and an angle of between thirty and fifty degrees increased the flow velocity compared to other angles in the lateral inflow channel.

Keywords: open channel; lateral inflow channel; finite difference; wetted perimeter.

1. Introduction

Open channels can either be constructed or natural conveyance for which water flows. The water is exposed to the atmosphere and water flows due to gravity. The geometry of the channels varies in size and shape. An open channel can either be rectangular, trapezoidal, circular or elliptic in shape.

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A lot of research has been done on open channels but few have been done on open channels with lateral inflow or outflow channel. Open channels have been with man for a long time. Man has always tried to design a channel that is efficient more so to help in drainage in flood-stricken areas and to convey water for irrigation purposes. Thus, the aim of this research is to come up with an efficient trapezoidal open channel that will meet these needs. Research on open channels with different cross sections has been extensively researched. The Chezy equation was the first equation that was developed to calculate the velocity of uniform flow. This velocity formula was first developed by the research study [1] on flow in open channels. Furthermore, this study was the first to pioneer various terms and definitions of open channel flow geometric elements. The earliest equation that was developed to compute the velocity of uniform channels was by the study [2]. However, since this equation does not take into account the coefficient of roughness which varies from one channel to another due to factors like the materials used to make the channel, the manning formula is used since it incorporates this coefficient [3]. For uniform lateral inflow, the research study [4] was able to formulate the diffusive equation for a wave. In this research, they were able to formulate the continuity and momentum equations for an open channel that had a lateral inflow channel joining this main open channel at an angle. The research study [5] were able to investigate how the lateral intake angle affects the discharge ratio of lateral intakes at one hundred- and eighty-degrees bend. The investigations that they carried out were done with clean water in a laboratory flume. They found out that at lateral intakes of forty-five degrees, the discharge ratio increased at all locations bends of the one eighty degrees flume. Yang [6] also in his research discovered that a diversion angle of between thirty and forty-five degrees gave a better flow pattern of the fluid when he studied flow structures with diversion angles of between zero and ninety degrees. The research study [7] was able to compare fluid flow in a rectangular and trapezoidal channel. It was found out that the graphs of trapezoidal cross-section channel showed higher values compared to rectangular cross-section channel of the flow velocity. The research study [8] was able to investigate fluid flow in an open rectangular channel with lateral inflow channel. It was established that increasing the area and length of the lateral inflow channel led to a decrease in the flow velocity in the main channel. Moreover, increasing the velocity of fluid in the lateral inflow channel led to an increase in the fluid flow velocity in the main channel. However, an angle of between thirty and fifty degrees of the lateral inflow channel led to higher fluid flow velocity in the main channel compared to other angles. According to the research [9], Chagas and Souza studied rivers by modelling equations that were able to simulate the flood wave. They used the Saint-Venant equations and showed in their research that hydraulic parameter played an important role in flood wave. This research was able to discretise the Saint-Venant equations using the finite difference method. The research [10] on modelling of open channels with circular cross section, discretised the equations governing the flow using the finite difference method and solved the equations simultaneously. Thus the finite difference method is an appropriate numerical tool to solve the equations governing the flow of the fluid. For any civilization to exist and thrive, it needs water. Water is life, yet too much water can lead to death due to floods. To direct water to lakes and rivers, man has constructed channels and canals. However, the problem of flood still persists, especially when there is heavy rain. Up to date, there is still a challenge to construct a channel that has a lateral inflow channel that will convey the maximum amount of water in an efficient way. Therefore, an efficient model of open channels with lateral intake channels has to be designed to meet these needs. The mathematical model in this study can be employed in the construction of lateral inflow channels that will increase the discharge while conveying water to farms for irrigation and in draining water from flood-stricken areas. The research will also help in construction of drainage in road designs to avoid poor drainage. Moreover, the findings are applicable in flour or textile production and in the design of water mills where large volumes of high velocity water are required to turn large turbines and also drive mechanical processes.

2. Hypothesis

The following assumptions are considered when deriving the continuity and momentum equations governing the flow in the open trapezoidal channel with lateral inflow channel.

- The flow is one-dimensional such that the main component of velocity is along the x-axis and is a function of x alone
- The forces causing the flow are due to gravity alone
- The fluid is considered incompressible
- The fluid is Newtonian
- The flow is unsteady
- Sediment formation between the lateral inflow channel and the main open channel is negligible
- Turbulent formation between the lateral inflow channel and the main open channel is negligible.

3. Governing Equations

Figure 1 illustrates the geometric model of the trapezoidal channel with the trapezoidal lateral inflow channel at an angle. Figure 2 illustrates the geometric model close up of the trapezoidal open channel and the trapezoidal lateral inflow channel. The discharge in the open trapezoidal channel and the lateral inflow channel will be denoted by Q and q respectively, while θ and L will represent the varying angle and length respectively of the lateral inflow channel. The net volume of fluid is represented by *dx* which is considered at an interval time of *dt*.

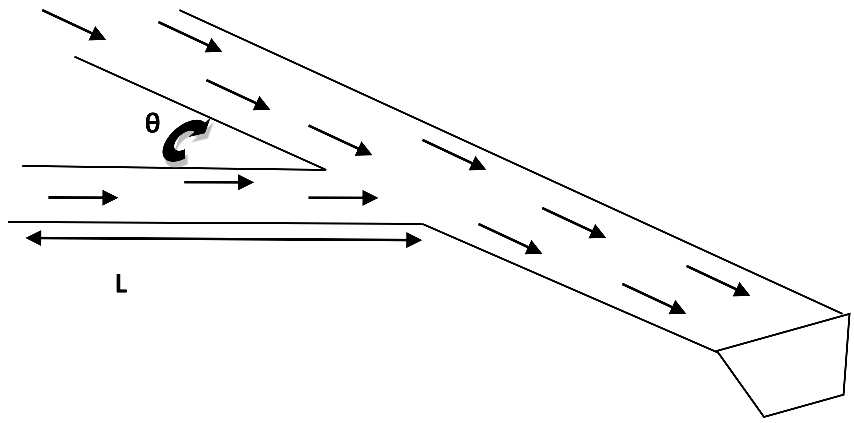

Figure 1: geometry of the model

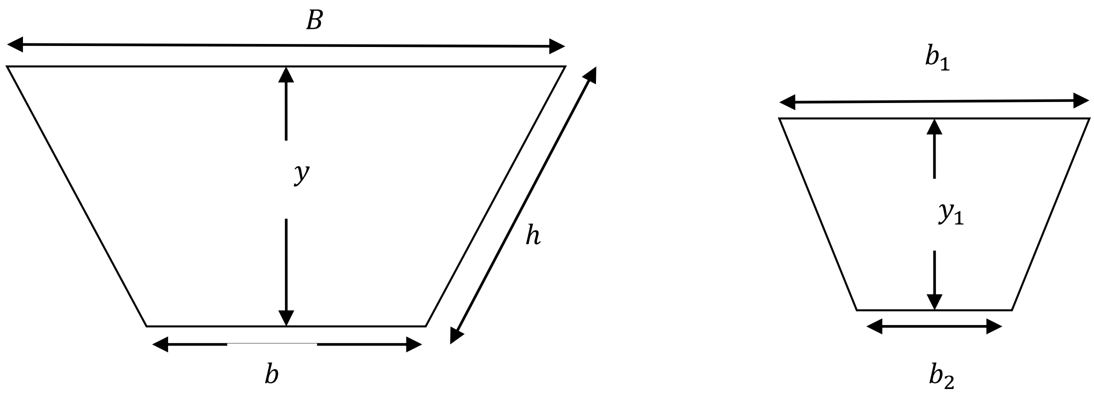

Figure 2: close up of main trapezoidal channel and lateral inflow channel respectively

The cross-sectional area of a trapezoidal channel is given by

$$A = y * \frac{1}{2}(B + b) \tag{1}$$

where A is the cross-sectional area, y the height, B and b the corresponding width of the main trapezoidal channel as shown from Figure. 2.

The wetted perimeter of the same channel is,

$$P = 2h + b \tag{2}$$

where P denotes the wetted perimeter and h the length of the side of the sides of the main trapezoidal channel.

The discharge of the main trapezoidal channel is given by

$$Q = A.V \tag{3}$$

where Q denotes the discharge, A the cross-sectional area and V the velocity of the main trapezoidal channel.

The discharge of the lateral trapezoidal channel is given by

$$q = a.u \tag{4}$$

where q denotes the discharge, a the cross-sectional area and u the velocity of this lateral inflow channel

The area of this lateral channel is given by

$$a = y_1 * \frac{1}{2}(b_1 + b_2) \tag{5}$$

where y_1 dotes the height and b_1 and b_2 the corresponding width as shown from Figure 2 of the lateral inflow channel.

The governing equations are the continuity and momentum equations which are given respectively by [11]. These two equations have been modified to incorporate a lateral inflow channel at an angle that drains into the main open channel.

$$\frac{\partial y}{\partial t} + V\frac{\partial y}{\partial x} + \frac{A}{T}\frac{\partial V}{\partial x} = \frac{q}{T\,L}\sin\theta \tag{6}$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} + g\left(S_f - S_0\right) = \frac{q}{A\,L}\sin\theta\ \left(u\cos\theta - V\right) \tag{7}$$

4. Method of solution

Equations (6) and (7) are nonlinear partial differential equations of first order. These equations cannot be solved analytically hence the finite different method will be used to solve them numerically [12]. The partial derivatives are replaced by finite difference approximations which result to a system of equations which are coupled together and solved by Matlab software [13]. Now converting these equations (6) and (7) to finite difference approximation discretization

$$\begin{aligned} y(i,j+1) = 0.5\big(y(i-1,j) + y(i+1,j)\big) - \triangle t\Bigg\{ & V(i,j)\frac{y(i+1,j)-y(i-1,j)}{2\triangle x} \\ & + \frac{A}{T}\frac{V(i+1,j)-V(i-1,j)}{2\triangle x} - \frac{q}{T\,L}\sin\theta\Bigg\} \end{aligned} \tag{8}$$

$$\begin{aligned} V(i,j+1) = 0.5\big(V(i-1,j) + V(i+1,j)\big) - \triangle t\Bigg\{ & V(i,j)\frac{V(i+1,j)-V(i-1,j)}{2\triangle x} \\ & + g\frac{y(i+1,j)-y(i-1,j)}{2\triangle x} \\ & + g\left[\frac{n^2}{2R^{\frac{4}{3}}}\left(V^2(i-1,j)+V^2(i+1,j)\right) - S_0\right] - \frac{q}{A\,L}\sin\theta\ \left(u\cos\theta - V(i,j)\right)\Bigg\} \end{aligned} \tag{9}$$

To complete the model, a set of initial and boundary conditions are introduced. The initial conditions are

$V(x,0) = 20\ and\ y(x,0) = 4.5\ for\ x > 0$

The boundary conditions depend on the characteristics of fluid flow are given by

$V(0,t) = 20\ and\ y(0,t) = 4.5\ for\ t > 0.$

The following values parameters were taken as follows,

$g = 9.81, S_0 = 0.002, B = 2, b = 1,\ n = 0.012\ and\ R = \frac{A}{R}$

The other parameters are varied one a time and investigations discussed in the next section. Equations (8) and (9) are solved together with the initial and boundary conditions then simulated by Matlab software.

5. Results and Discussion

Matlab computer software was used to solve and plot the various graphs for the equations (8) and (9) at specific points along the channel. To get the various graphs below, various velocity profiles were plotted at a specific point along the main channel and various parameters varied independently as the others remained a constant. Figure 3 and Figure 5 were plotted at a constant angle of forty degrees of the lateral inflow channel. Various conclusions on how the surface area, length of the channel, velocity of flow and angle of the lateral inflow channel affect the velocity in the main trapezoidal channels are then presented.

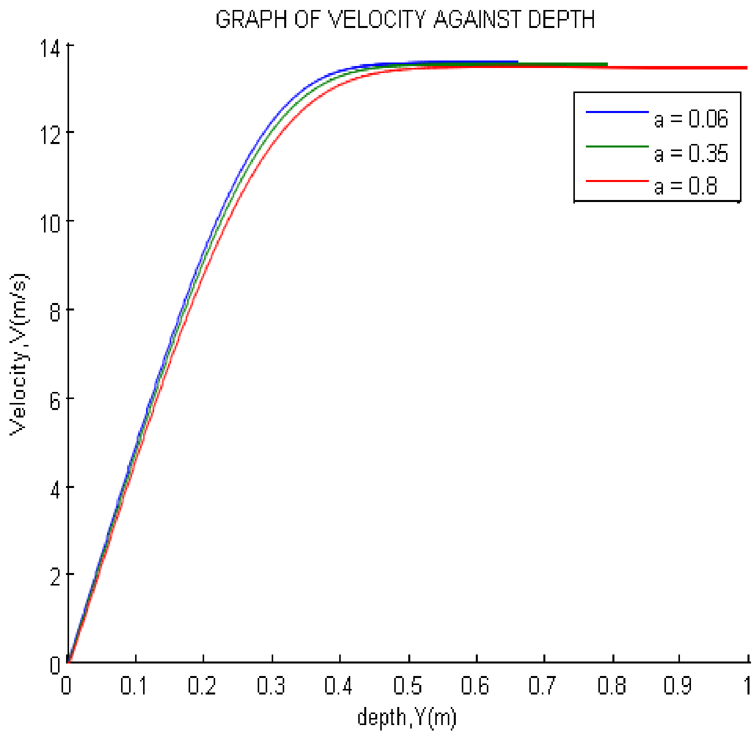

Figure 3: Velocity depth graph for varying cross-section area

From figure 3, it is noted that the velocity of the fluid increases with depth with maximum of 4.5m. As the depth increases, the velocity is noted to remain constant. The velocity is constant at depths more than 4.5m because of the resistance of flow at the free surface. Moreover, increasing the cross-sectional area of the trapezoidal lateral inflow channel from 0.06 m^2 to 0.8 m^2 leads to a decrease in the velocity of the fluid in the main trapezoidal channel. The reason is due to the fact that increasing the area of this channel leads to an increase in the wetted perimeter. The increase in the wetted perimeter leads to increase in the shear stress or in other words the fluid covering a more surface area in the channel due to increase in the width of the channels which leads to more resistance from the sides of this channel. This resistance on the walls of the channel contributes to the reduction of the velocity in the channel.

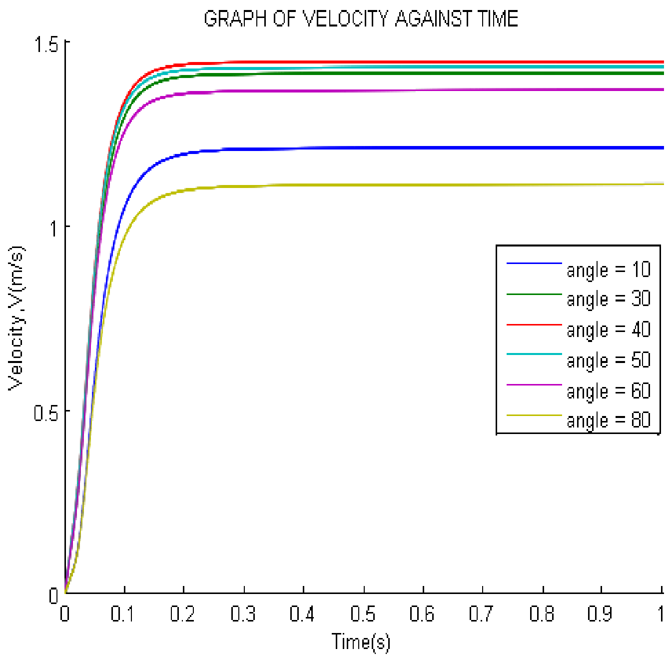

Figure 4: Velocity time graph for varying lateral inflow angle

From figure 4, increasing the angle of the trapezoidal lateral inflow channel in relation to the main trapezoidal channel has some interesting findings. It is found out that an angle of forty degrees exhibits higher velocity in the main trapezoidal channel than any other angle. An angle of fifty degrees closely follows. At these angles the resistance of flow at the entrance of the trapezoidal is lower than the other angles hence higher velocity in the main channel. Since in the construction of this lateral inflow channel one cannot be perfect, it is recommended that if one wants to get the maximum discharge from this channel, an angle of between thirty and fifty degrees will suffice.

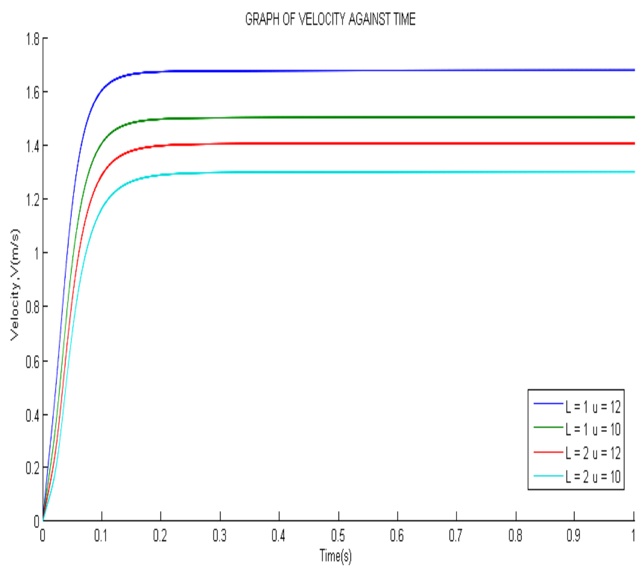

Figure 5: Velocity time graph for varying lateral inflow length

From figure 5, increasing the length of the channel from 1 m to 2 m of the trapezoidal lateral inflow channel leads to a decrease in the flow velocity of the main trapezoidal open channel. The reason for this is due to the fact that increasing this length leads to an increase in the shear stress of the lateral inflow channel on its wall and bottom which led to a reduction in the velocity. However, increasing the flow velocity of this lateral inflow channel from 10 m/s to 12 m/s leads to an increase in the flow velocity in the main trapezoidal channel. The reason for this is due to more bombardments of the fluid particles with one another which increases the kinetic energy of the fluid particles which in turn increases the velocity the whole fluid.

6. Conclusions

A mathematical model of fluid flow in a trapezoidal channel with lateral inflow channel has been developed. The continuity and momentum equations governing the flow which have been discretised using the finite difference approximation method and solved. Then various velocity profiles graphs have been obtained which show how the cross-sectional area, angle, length and velocity of the trapezoidal lateral inflow channel affect the flow velocity in the main trapezoidal channel. The results of this research establish that the velocity of flow increase with depth of the channel and an increase in the cross-sectional area of the channel leads to a reduction in the flow velocity. That an angle of forty degrees of the lateral intake channel exhibits higher velocity and increase in length of the lateral inflow channel leads to a reduction in the overall velocity of the main channel. Finally, the flow velocity in the main channel increases with increase in velocity of the lateral inflow channel. These results of this research agree well other related research in open channel flow.

7. Recommendations

The research assumed one dimensional fluid flow where thermal effects were not considered. The model could have been more realistic if two-dimensional flow together with the energy equation were considered. It is the recommendation of this study that future research should concentrate on how sediments and contaminants affect open channel flow.

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